Five-Year Integrated M. Sc. Examination 2021-2022

Semester: V

Paper: MT-3-5-4

Subject: Complex Analysis

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

1. a) Define a bilinear transformation. Find the fixed points of the bilinear transformation $w=\frac{1+iz}{i+z}$.

b) Find the bilinear transformation which maps the points $z_1=1, z_2=i, z_3=-1$ respectively into the points $w_1=0, w_2=1, w_3=\infty$.

c) If the bilinear transformation $w=\frac{a+bz}{c+dz}, ad-bc\neq 0$ has two finite fixed points α, β then show that it can be expressed as $\frac{w-\alpha}{w-\beta}=k\frac{z-\alpha}{z-\beta}$, where k is a constant. [4+3+3]

2. a) State Cauchy Integral Theorem.

Evaluate $\oint_C \frac{e^{2z+1}}{(z+1)^{-4}}dz$, where C is the circle C : |z-1| = 1.

b) State Cauchy Integral formula.

Evaluate $\oint_C \frac{2z-1}{z^2-z}dz$, where C is a simple closed curve containing 0 and 1 in its interior.

c) Define zero of an analytic function. [4+4+2]

3. a) State and prove Riemann's theorem on removable singularity.

b) When a function is said to have a pole at α of order m.

c) If f(z) has a pole at α, then prove that $|f(z)|\to\infty$ as $z\to\alpha$ in any manner. [5+2+3]

4. a) State and prove Laurent's theorem.

b) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in a Laurent series in the region 1<|z|<3. [7+3]

5. a) Let α be a pole of f(z) of order m and a pole of g(z) of order n (< m). Find the order of the pole with respect to f(z)/g(z).

b) If a function $f(z)$ is regular in a domain D and if $z_1, z_2, \ldots\ldots, z_n, \ldots.$ is a sequence of zeros of $f(z)$ having as limit point an interior point α of D, then $f(z)\equiv 0$.

c) Show that the sum of the residues of $\frac{e^{-z}}{z^2-a^2}$ at its poles is $-\frac{1}{a}\sinh a$. $[3+4+3]$

6. a) State and prove Cauchy's residue theorem.

b) Using Cauchy's residue theorem show that $\oint_{|z|=1}\frac{e^{2z}}{\cosh \pi z}dz=4i\sin 1$. [6+4]